Chained Optimization Scheme for Simultaneous Co-allocation of Resources of Different Types

Authors:

Victor Toporkov
Dmitry Yemelyanov

Department of Computing Technologies
National Research University “Moscow Power Engineering Institute”
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Outline

• Parallel jobs scheduling in high performance and cloud systems requires the use of significant resources of different types (computational nodes, data storages, software packets, network connections, etc.)

• Known scheduling algorithms are designed to allocate homogeneous or heterogeneous resources of the same type

• We propose a solution for the problem of simultaneous co-allocation of resources of different types
Single-type Resource Requests

The resource requirements for a single job execution are arranged into a resource request:

- \( n \) - number of simultaneously reserved computational nodes
- \( \rho \) - minimal performance requirement for each computational node
- \( V \) - computational volume for a single node task
- \( C \) - maximum total job execution cost (budget)

Heterogeneous resources of the same type share the same characteristics:

- performance and cost for computing nodes
- volume and cost for memory modules
- storage capacity and cost for SSD
- etc.

\[
T = \frac{V}{\rho_{\text{min}}}
\]

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Window Search Problem

Allocate a window of computing four nodes for a time $T$, with requirements on nodes performance and total cost. Minimize window start time:

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0-1 Knapsack Slots Subset Allocation

Candidate resources

| 1  | C1, Z1 |
| 2  | C2, Z2 |
| ... |   ... |
| m  | Cm, Zm |

\[ Z = \sum_{i=1}^{m} z_i x_i \rightarrow \max \]

\[ \sum_{i=1}^{m} c_i x_i \leq c_j, \]

\[ x_i \in \{0,1\}, i = 1, \ldots, m \]

Allocated resources

| 1  | Ci, Zi |
| 2  | Cj, Zj |
| ... |   ... |
| n  | Ck, Zk |

Number \( n \) of allocated resources is not limited: \( n \in [0; m] \)

Classic 0-1 knapsack problem
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Limited Size Slots Subset Allocation

Candidate resources

1  C1, Z1
2  C2, Z2
...  ...
1  Cm, Zm

\( n \leq m \)

Allocated resources

1  Ci, Zi
2  Cj, Zj
...  ...
n  Ck, Zk

Number \( n \) of simultaneously required resources is predetermined

\[
Z = \sum_{i=1}^{m} z_i \ x_i \rightarrow \max
\]

\[
\sum_{i=1}^{m} c_i \ x_i \leq c_j,
\]

\[
\sum_{i=1}^{m} x_i = n,
\]

\( x_i \in \{0,1\}, i = 1, \ldots, m \)
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**Interval-based Slots Subset Allocation**

Candidate resources

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C1, Z1</td>
</tr>
<tr>
<td>2</td>
<td>C2, Z2</td>
</tr>
</tbody>
</table>
| ... | ...
| m | Cm, Zm |

n ≤ m

Allocated resources

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ci, Z1</td>
</tr>
<tr>
<td>2</td>
<td>Cj, Zj</td>
</tr>
</tbody>
</table>
| ... | ...
| n | Ck, Zk |

\[
Z = \sum_{i=1}^{m} z_i x_i \rightarrow \max
\]

\[
\sum_{i=1}^{m} c_i x_i \leq C,
\]

\[
\sum_{i=1}^{m} x_i \geq n_{\min},
\]

\[
\sum_{i=1}^{m} x_i \leq n_{\max},
\]

\[
x_i \in \{0,1\}, i = 1, \ldots, m
\]

Interval of permissible values \([n_{\min}; n_{\max}]\) is defined for \(n\)

This problem describes a more generic resources allocation scenario
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Recurrent Solution for Interval Problem

\[ f_i(c, k) = \max\{f_{i-1}(c, k), f_{i-1}(c - c_i, k - 1) + z_i\}, \]

\[ i = 1, \ldots, m, c = 1, \ldots, C_j, k = 1, \ldots, n_{\text{max}} \]

\[ Z_{\text{max}} = \max_n f_m (C, n) \]

\[ O(m \times n_{\text{max}} \times C) \]
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Composite Scheduling Model

- Different allocation requirements are given for different type of the resources

- There is usually a single common constraint on the total resources’ usage price: a budget allocated for the job execution

The united scattered requirements may be compiled in a single composite resource request, for example:

- allocate \( n_1 \) computational nodes, the corresponding number \( n_2 \) of software packets, \([n_3;n_4]\) storage drives and at least one network connection

- total allocation budget \( C \)

- perform optimization by the criterion function \( Z \).
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Chain Algorithm

The recurrent calculations for sub-problem $G_j$ is passed as an input initialization data for sub-problem $G_{j+1}$

Vector $f_0(c, 0), c = 0, \ldots, C$ determines the best attainable solution for all the previous sub-problems

Resulting solution when the recurrent scheme for the last $N$th problem is calculated:

$$Z_{\text{max}} = f_{m_N}^N(C, n_N).$$
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Resources Allocation Example

Composite request:

allocate 8-12 VMs, 1-8 storage drives and one OS delivery and support plan (up to 15 workstations) for 350 dollars a month

maximize $\sum_{i=1}^{m} z_i$, where $z_i$ is pre-calculated user utility estimation of each offer

Solution:

7 VMs with 2GB RAM for 10 dollars, one VM with 1GB RAM for 5 dollars, three additional storage drives of 500Gb each and five drives of 250 Gb each, free OS distribution

<table>
<thead>
<tr>
<th>Type</th>
<th>Vendor</th>
<th>Characteristic</th>
<th>Price, $c_i$</th>
<th>Utility, $z_i$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Vendor 1</td>
<td>1Gb RAM</td>
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<td>8Gb RAM</td>
<td>62</td>
<td>8</td>
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<td>61</td>
<td>8</td>
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<td>Vendor 1</td>
<td>250Gb</td>
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<td>8</td>
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<tr>
<td>DISC</td>
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<td>500Gb</td>
<td>50</td>
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<tr>
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<td>Vendor 1</td>
<td>1000Gb</td>
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<td>25</td>
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<td>OS</td>
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<td>OS3 Professional Support</td>
<td>1000</td>
<td>20</td>
</tr>
</tbody>
</table>
Simulation Study

- Heterogeneous environment consists of:
  - VMs,
  - storage drives
  - software products

- Single composite resource request:
  - allocate 8-12 VMs, any number of storage drives and one (common) license for a software product
  - execution budget is 350

- Utility values $z_i$ precalculated randomly for each of 200 different resource instances
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Greedy Solution GX

\[ \sum C_i = C \]

1. GX distributes the available budget \( C \) between the individual sub-problems
2. Each sub-problem is solved independently by the corresponding greedy algorithm
3. The results of the individual solutions are combined into a common composite solution

Budget distribution strategies:

1. **GUniform** distributes budget equally: \( C / N \) for each sub-problem
2. **GProportional** allocates budget \( C \) according to a heuristic metric proportionally to the sub-problem requirements
3. **GReferenced** accepts the composite problem solution obtained by the **Chain** algorithm as a reference and distributes the available budget in the same proportion (we assume that **Chain** algorithm provides the optimal budget distribution)
Considered Algorithms

For the experiment simulation we consider the following algorithms.

Feasible algorithms:

- \textit{Chain} knapsack-based algorithm
- \textit{GUniform} greedy algorithm
- \textit{GProportional} greedy algorithm

Infeasible algorithms (for additional evaluation):

- \textit{GReferenced} greedy algorithm (requires \textit{Chain} solution as an input)
- \textit{GUniform X2.1} greedy algorithm with available budget 2.1 times increased
- \textit{GProportional X1.35} greedy algorithm with available budget 1.35 times increased
- \textit{GReferenced X1.18} greedy algorithm with available budget 1.18 times increased
### Simulation Results

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$Z_{\text{max}}$</th>
<th>Used budget</th>
<th>Working Time, ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>GUniform</td>
<td>622.9</td>
<td>220.2</td>
<td>0.4</td>
</tr>
<tr>
<td>GProportional</td>
<td>752</td>
<td>322.5</td>
<td>0.4</td>
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<tr>
<td>GReferenced</td>
<td>776.9</td>
<td>323.2</td>
<td>0.4</td>
</tr>
<tr>
<td>GUniform X2.1</td>
<td>825.6</td>
<td>387.4</td>
<td>0.4</td>
</tr>
<tr>
<td>GProportional X1.35</td>
<td>829.9</td>
<td>391.8</td>
<td>0.4</td>
</tr>
<tr>
<td>GReferenced X1.18</td>
<td>826.6</td>
<td>380.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Chain</td>
<td>827.4</td>
<td>349.8</td>
<td>34.3</td>
</tr>
</tbody>
</table>

Results were obtained based on 6000 independent randomized experiment scenarios.

Each scenario included resources allocation problem in an environment with 3 types of resources and $C = 350$ $\$ budget limit.
Chained Optimization Scheme for Simultaneous Co-allocation of Resources of Different Types

Simulation Results

- **Chain** knapsack-based algorithm advantage by the target criterion Z over greedy analogues ranges from 6% to 25%

- **Greedy** algorithms require 35% to 110% additional budget to catch up with the chain solution

- **Chain** algorithm requires much more time for the calculations (100x times in our test environment)

- **GProportional** algorithm with budget distribution can outperform greedy analogues
Conclusion and Future Work

- **Chain** algorithm is proposed to use knapsack-based allocation procedure for composite scheduling problems with resources of different types (computational nodes, data storages, software packets, network connections, etc.) and a common cost limit.

- **Chain** algorithm *automatically* solves the problem of proper budget distribution between the constituent subproblems.

- This allows the Chain algorithm to noticeably outperform greedy analogues and justify its relatively high computational complexity.

- The future research directions include design and testing of additional technics for the chain algorithm complexity optimization.
Chained Optimization Scheme for Simultaneous Co-allocation of Resources of Different Types

References


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